Dr. Julian Detemple Johannes Gutenberg-University Mainz, Summer 2025

GAME THEORY AND STRATEGIC DECISION MAKING

– EXAM –

July 28, 2025

Please carefully read the following before you start working on the exam:

- Please write your matriculation number on each page of both question and answer sheet.
- **Write all your answers on the separate answer sheet.**
  Make all your calculations on the answer sheet.
- You may scribble on the question sheet, **but only what is on the answer sheet will be graded**.
- When handing in your exam, please enclose this question sheet inside the answer sheet.

- A total of **60 points** can be achieved. Scores for individual questions are noted to their right.
- When noting your answers, please pay attention to formal correctness – i.e. if asked for a set, please state a set.
- You may use all notational conventions as introduced in lecture and tutorial.
- The use of additional tools other than a calculator is **not allowed**.

Good luck!

1 **(7 points total)**

Consider a Cournot duopoly game where two firms simultaneously set their quantities. The symmetric firms both have production costs of 6 per produced unit. The market price is given by $p = 24 - q_1 - q_2$.

a) What are the firms profit functions? (2 points)

b) Derive the firms' best responses. (3 points)

c) Find the Nash Equilibrium of this game. (2 points)
If you could not solve task b), please assume the following best response functions of the firms: $BR_1(q_2) = 12 - \frac{1}{3}q_2$, $BR_2(q_1) = 12 - \frac{1}{3}q_1$.

2

(11 points total)

Consider the following normal form game:

		P2			
		W	X	Y	Z
P1	A	1, 7	3, 3	7, 2	0, 5
	B	4, 1	3, 2	4, 4	2, 3
	C	3, 6	7, 4	8, 3	5, 4
	D	6, 2	2, 5	1, 3	1, 2

a) State the following payoffs:

(i) $u_1(B, Y)$ (0.5 points)

(ii) $u_2((\frac{1}{4}, \frac{3}{4}, 0, 0), Z)$ (0.5 points)

(iii) $u_2((0, \frac{1}{2}, 0, \frac{1}{2}), (\frac{1}{2}, 0, 0, \frac{1}{2}))$ (1 point)

b) State the pure strategy spaces of the players. (1 point)

c) Find best responses for the following beliefs:

(i) $\theta_2 = (1, 0, 0, 0)$ (1 point)

(ii) $\theta_1 = (0, \frac{1}{2}, \frac{1}{2}, 0)$ (1 point)

d) State the set of efficient strategy profiles. (2 points)

e) Find the set of rationalizable strategies. (4 points)
Please, document your steps: whenever you eliminate a strategy, clearly state the reason.

3 **(11 points total)**

Consider the following normal form game:

		P2	
		a	b
P1	A	8, 3	2, 1
	B	4, 4	3, 8

a) Find all Nash Equilibria of this game. Show your steps. (5 points)

b) Now, assume that the payoffs in the game matrix only represent monetary payoffs, i.e. payments. Assume further that utility over these monetary payoffs is now defined by the following utility function of the players: $U_i(x_i, x_j) = x_i - \frac{1}{5}(x_i - x_j)^2$ where x_i represents player i's own monetary payoff and x_j the other players monetary payoff. Thus, people dislike inequality and do not only care about their own payout but also about the difference to the other player's payout. Find all Nash Equlibira in **pure strategies** under this assumption. Show your steps. (6 points)

4 **(7 points total)**

Consider the following repeated game:

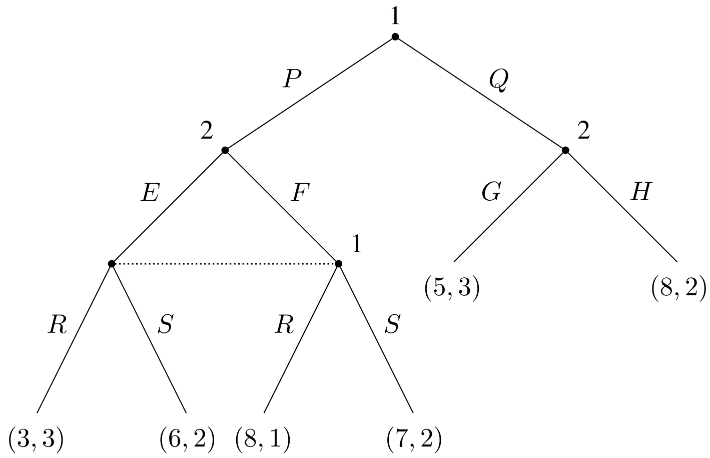
		P2: P	P2: N
P1	P	4, 5	-1, 17
	N	12, -1	0, 1

a) Is there an SPE with (P, P) played in period t=1 if T = 3? If yes, state under which condition(s). If not, briefly explain why. (2 points)

b) Assume player 1 has a discount factor of $\delta_1 = 0.8$ while player 2 has a discount factor of $\delta_2 = 0.6$. Under these conditions, is there an SPE that sustains (P,P) in every period for $T = \infty$, using the Grim Trigger Strategy as discussed in class (start with (P,P) but switch to (N,N) for ever after a single deviation)? Explain your answer. (5 points)
(Hint: Remember that $1 + \delta + \delta^2 + \delta^3 + ... = \sum_{i=0}^{\infty} \delta^i = \frac{1}{1-\delta}$*.)*

5 **(8 points total)**

Consider the following extensive form game:

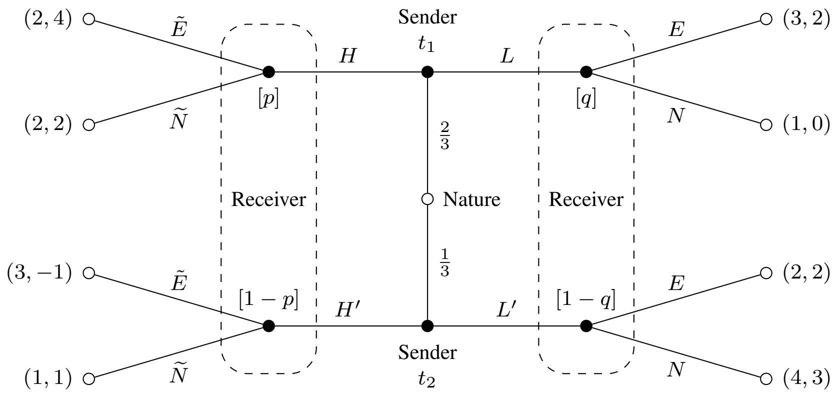


a) How many subgames does this game have? (1 point)

b) Write down the pure strategy spaces of both players. (2 points)

c) Find and state the subgame perfect equilibrium of this game. *(Note: Consider only pure strategies.)* (5 points)

6 **(16 points total)**



a) Does this game have any *separating* perfect Bayesian equilibrium? Show your **complete** analysis and if there is such an equilibrium, report it. (8 points)

b) Does this game have any *pooling* perfect Bayesian equilibrium? Show your **complete** analysis and if there is such an equilibrium, report it. (8 points)