Prof. Dr. Florian Hett Johannes Gutenberg-University Mainz, Summer 2024

GAME THEORY AND STRATEGIC DECISION MAKING

July 29, 2024

Please carefully read the following before you start working on the exam:

- Please write your matriculation number on each page of both question and answer sheet.
- **Write all your answers on the separate answer sheet.**
  Make all your calculations on the answer sheet.
- You may scribble on the question sheet, **but only what is on the answer sheet will be graded**.
- When handing in your exam, please enclose this question sheet inside the answer sheet.

- A total of **60 points** can be achieved. Scores for individual questions are noted to their right.
- When noting your answers, please pay attention to formal correctness – i.e. if asked for a set, please state a set.
- You may use all notational conventions as introduced in lecture and tutorial.
- The use of additional tools other than a calculator is **not allowed**.

Good luck!

1 **(19 points total)**

Consider the following normal form game:

		P2 K	P2 L	P2 M	P2 N
P1	A	6, 3	5, 7	3, 6	4, 5
	B	1, 2	3, 5	7, 8	0, 7
	C	4, 7	6, 7	4, 2	2, 8
	D	3, 7	5, 6	9, 5	1, 4

a) Please state the following payoffs: (2 points)

- $u_1(C, L)$
- $u_2\big((\frac{1}{2}, 0, \frac{1}{2}, 0), K\big)$

b) Write down the strategy spaces of both players. (2 points)

c) State the set of efficient strategy profiles. (2 points)

d) Find best response for the following beliefs: (2 points)

- $\theta_2 = (0, 1, 0, 0)$
- $\theta_1 = (0, \frac{1}{2}, 0, \frac{1}{2})$

e) Perform iterated elimination of dominated strategies. (4 points)
Please document your steps: whenever you eliminate a strategy, clearly state a reason; when you end the procedure, clearly state why.

f) What is R^∞ (the set of rationalizable strategies) of this game? (1 point)

g) Find Nash equilibria of this game. Show your analysis. (6 points)

2 (11 points total)

Consider the following normal form game:

		P2 L	P2 R
P1	U	6, -8	-2, 0
	M	5, -7	1, -3
	D	-1, -1	3, -5

a) Assume that Player 2 in the above game plays L with probability p and R with $1-p$. Construct a diagram with $p \in [0, 1]$ on the x-axis and u_1 on the y-axis; graph the payouts of Player 1's strategies U, M, and D conditional on p. (3 points)

b) For those strategies that are not strictly dominated, state to which beliefs they constitute best responses. (8 points)

3 **(14 points total)**

Consider the following stage game **to be repeated twice**:

		P2 L	P2 C	P2 R
P1	U	2, 2	6, 1	1, 1
	M	1, 6	5, 5	1, 1
	D	1, 1	1, 1	4, 4

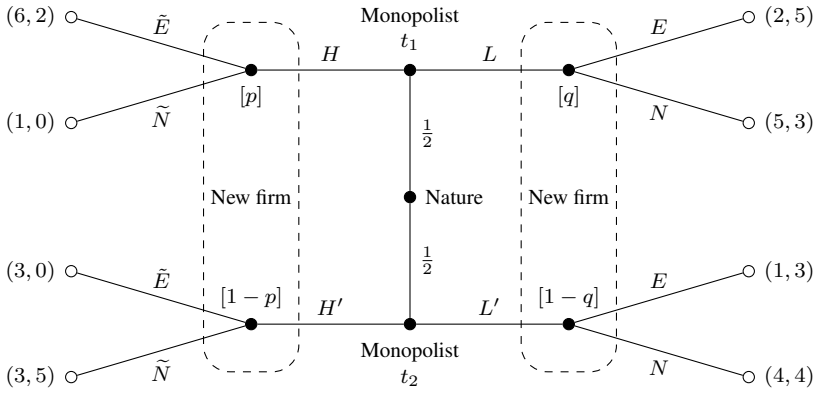
a) Find all pure strategy Nash equilibria of the stage game. (2 points)

b) Is there a subgame perfect equilibrium (SPE) in which (M, C) is played in $t = 2$? (1 point)

c) Is there a subgame perfect equilibrium (SPE) in which (M, C) is played in $t = 1$? (6 points)

d) Now suppose that this stage game is repeated for infinite time, i.e. $T = \infty$.
Can action profile (M, C) be a subgame perfect equilibrium?
Hint: assume that punishment after deviation is played forever. (5 points)

4 (16 points total)



a) Does this game have any *separating* perfect Bayesian equilibrium? Show your **complete** analysis and if there is such an equilibrium, report it. (8 points)

b) Does this game have any *pooling* perfect Bayesian equilibrium? Show your **complete** analysis and if there is such an equilibrium, report it. (8 points)